

# Period of the $\gamma$ -ray staggering in the $^{150}\text{Gd}$ superdeformed region

D. Samsoen,<sup>1</sup> P. Quentin,<sup>1</sup> and I. N. Mikhailov<sup>2,3</sup>

<sup>1</sup>*Centre d'Études Nucléaires de Bordeaux Gradignan, CNRS-IN2P3 and Université Bordeaux-I, F-33175 Gradignan, France*

<sup>2</sup>*Bogoliubov Laboratory of Theoretical Physics (Joint Institute of Nuclear Research), 141980 Dubna (Moscow region), Russia*

<sup>3</sup>*Centre de Spectrométrie Nucléaire et de Spectrométrie de Masse CNRS-IN2P3, 91405 Orsay, France*

It has been previously proposed to explain  $\gamma$ -ray staggerings in the deexcitation of some superdeformed bands in the  $^{150}\text{Gd}$  region in terms of a coupling between global rotation and intrinsic vortical modes. The observed  $4\hbar$  period for the phenomenon is suggested from our microscopic Routhian calculations using the Skyrme SkM\* effective interaction.

21.10.Re, 21.60.Ev, 21.60.Jz

This brief paper completes the theoretical investigation of the coupling between global rotation and intrinsic vortical modes proposed in Ref. [1] as a tentative explanation for the rather rare band staggering observed in the decay of some superdeformed bands. Indeed while such a phenomenon was first claimed in  $^{149}\text{Gd}$  [2], then in  $^{194}\text{Hg}$  [3], and possibly in the  $A \sim 130$  superdeformed region [4], its existence has been confirmed and extended to a couple of neighboring nuclei in the first case [5] and ruled out in the second case [6]. Various theoretical explanations have been proposed [7–11] besides the one which we have discussed in Ref. [1]. When making an attempt to describe such data, one should address the three following questions: (i) What is the mechanism at work? (ii) Why is this phenomenon so scarce and what makes it appear where it is observed ( $A \sim 150$ )? (iii) What is tuning the period of the staggering?

While some answers have been provided in our previous papers [1,12–14] to the two first questions, we aim here at addressing the third one. In [1], a staggering in transition energies within the yrast band was shown to appear in cases where the relevant collective energy is quadratic in two quantized quantities. A particular realization of the latter, well suited to the description of fastly rotating superdeformed states, corresponds to the parallel coupling of global rotation and intrinsic vortical modes in ellipsoidally deformed bodies, known after Chandrasekhar [15] as  $S$  ellipsoids. In this case, the two commuting operators are the projections on the quantification axis of the angular momentum operator and of the so-called Kelvin circulation operator (see, e.g., Ref. [16]), hereafter called  $I$  and  $J$ , respectively. Whereas it is trivial to show that the Kelvin circulation operator satisfies the usual commutation relations of an angular momentum, its consideration as a quantity which is approximately a constant of the motion is a basic assumption of our collective model. Its exact amount of violation would deserve a specific microscopic study. A self-consistent description of such a coupling can be made upon generalizing the Routhian approach [17], amounting thus to solve the following variational problem:

$$\delta\langle H - \Omega I - \omega J \rangle = 0, \quad (1)$$

where  $H$  is the microscopic Hamiltonian,  $\Omega$  ( $\omega$ ) an angu-

lar velocity associated with the global rotation (intrinsic vortical) mode. This approach was first investigated in Ref. [12] within a simple oscillator mean field approximation. There, the assumption of an energy which is quadratic in  $(\Omega, \omega)$  or equivalently in  $(I, J)$  has been shown to be rather well satisfied. Recently, fully self-consistent solutions of the above variational problem have been made possible [18] upon using the standard SkM\* force [19]. Details about such calculations and some of their results will be discussed below.

In another paper [13], a physical analogy stemming from the well-known similarity between the motion of a charge in a magnetic field and of a mass in a rotating frame has been established. It relates this staggering phenomenon with the observation of persistent currents in mesoscopic conductor or semiconductor rings as a manifestation of an inherent Aharonov-Bohm phase. Apart from its obvious physical appeal this consideration has set a framework in which one is able to understand the scarcity of both phenomena. It results from the necessity of securing a sufficiently low level of a specific damping which happens to yield in the staggering case a condition on the width of the superdeformed state in the relevant collective variable (e.g., the usual axial quadrupole deformation  $\beta$ ). It was deduced from microscopic calculations of the associated mass parameters [14], using the usual D1S Gogny effective force [20], that such a condition of existence was generally not met for Ce and Hg superdeformed states as well as for such states in Gd isotopes but for  $^{150}\text{Gd}$  and possibly  $^{148}\text{Gd}$ .

As a consequence of all these studies the explanation for the staggering phenomenon suggested in Ref. [1] is of course neither exempted from *a priori* questions nor deemed as being the only possible one, yet it is conformed as a rather likely candidate. However, one point remains to be clarified concerning the  $4\hbar$  period of the staggering. In Refs. [1,12] this particular period was associated with a ratio of  $I$  and  $J$  values close to 2 for the considered states. From either semiclassical estimates of the relevant inertia parameters [1] or from actual microscopic calculations in the harmonic oscillator mean field approximation [12] such a ratio is much more consistent with hyperdeformation than with the actual superdeformation of the nuclear states. It is on this point that the new self-consistent ap-

proach of Ref. [18] brings some interesting insight.

Here the generalized Routhian variational problem is solved within the Hartree-Fock approximation. Numerical codes breaking the time-reversal and axial symmetries as requested by the considered physical problem, determine the single-particle wave functions either at the nodes of a spatial mesh [21] or as resulting from an expansion on a suitably chosen truncated basis. In the latter case, all approaches so far [22–24], to the best of our knowledge, have used a triaxial basis. In our calculations an alternative method, shown to be less time consuming in most cases, has been developed where the expansion is made on an axial basis. Here the dependence of various fields and densities in terms of the angular variable of the cylindrical coordinate system is handled by convenient Fourier expansions.

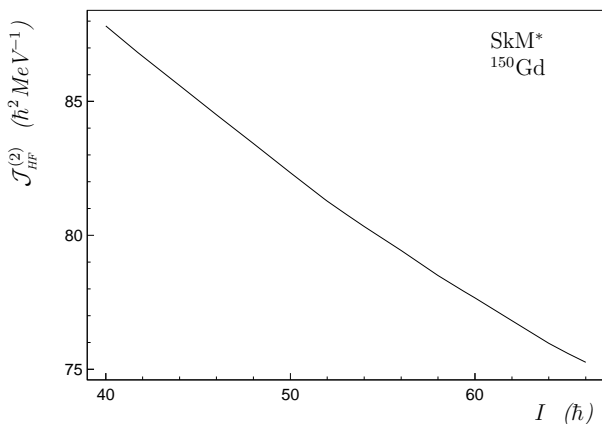


FIG. 1. Dynamical moment of inertia  $\mathcal{J}^{(2)}$  as a function of the angular momentum  $I$ .

Self-consistent solutions of the usual Routhian problem (i.e. without constraint on the Kelvin circulation, namely, for  $\omega = 0$ ) have been first performed for the  $^{150}\text{Gd}$  nucleus. As usual to get a solution corresponding to a given (quantized) value of  $I$ , one solves the problem iteratively so as to determine the angular velocity constraint  $\Omega$  able to reach the requested angular momentum. For such solutions, one can estimate the dynamical moment of inertia by numerical differentiation. As suggested in Ref. [25], among the possible expressions for  $\mathcal{J}^{(2)}$ , we choose for numerical stability reasons to evaluate

$$\mathcal{J}^{(2)} = \frac{\partial I}{\partial \Omega}. \quad (2)$$

Such derivatives have been determined from a two-point formula involving thus for each studied value of the angular momentum three different Hartree-Fock calculations (differing typically around  $I = 50\hbar$  by  $\Delta\hbar\Omega \sim 0.07$  keV). As discussed, e.g., in Ref. [12], even in the absence of any constraint on the intrinsic vortical currents, the Kelvin circulation operator takes a finite value which is easily estimated from our solutions. The vector components of

this operator are defined (see, e.g., Ref. [16]) by its action on single-particle wave functions as

$$J_\alpha = \frac{\hbar}{i} \sum_{\beta,\gamma} \epsilon_{\alpha\beta\gamma} \frac{c_\gamma}{c_\beta} x_\beta \frac{\partial}{\partial x_\gamma}, \quad (3)$$

where  $\epsilon_{\alpha\beta\gamma}$  is the completely antisymmetrical third-rank tensor,  $x_\alpha$  the  $\alpha$  component of the particle position vector, and  $c_\alpha$  the corresponding length scale factor. These factors are deduced from the values of the quadrupole tensor calculated from our variational solutions upon making an ellipsoidal shape approximation.

Some results of our Routhian calculations ( $\omega = 0$ ) for a range of  $I$  values from  $I = 40\hbar$  to  $I = 66\hbar$ , including states of relevance for the observed superdeformation, are displayed in Fig. 1. It shows the variation of the moment of inertia  $\mathcal{J}^{(2)}$  as a function of  $I$ . As a matter of fact, it fits rather well, as it should, with the pure Hartree-Fock part of the results obtained in Ref. [26] with the same interaction. The calculated yrast value  $J_{\text{yrast}}(I)$  of the Kelvin circulation as a function of  $I$  falls very nicely on the straight line:

$$J_{\text{yrast}}(I) \simeq 0.8I + 1.0\hbar. \quad (4)$$

This is not at all surprising insofar as the quadratic approximation for the collective energy discussed in Ref. [12] is valid over the whole considered range of values of  $I$ . However, this relation makes it very clear that around  $I = 50\hbar$ , e.g., the ratio  $I/J$  is indeed very far from the value of 2 which would lead to the observed staggering period.

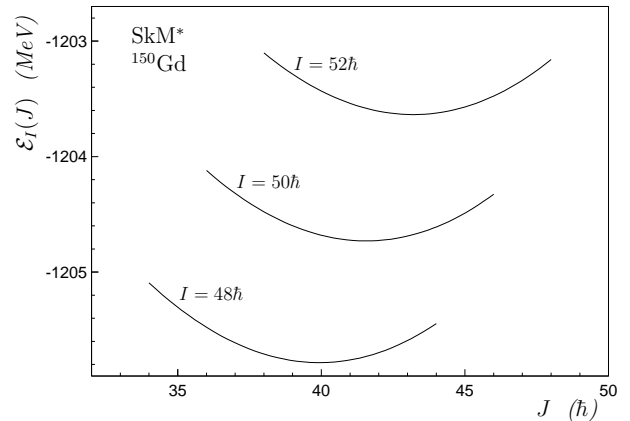


FIG. 2. Total energy  $\mathcal{E}_I(J)$  as a function of the Kelvin circulation  $J$  for angular momentum values around  $I = 50\hbar$ .

As a result of the continuity equation, tangential intrinsic vortical excitations yield phase-space modifications amounting only to a momentum redistribution [12]. In that respect they are indeed quite similar to pairing correlations. This has been, for instance, illustrated from another point of view in Ref. [27]. There, current patterns as functions of a fixed pairing gap display indeed

the same type of variation as classical  $S$ -ellipsoid velocity fields with respect to the angular velocity of the intrinsic vortical modes. Therefore one may consider the latter modes as a collective model translation of pairing correlations in a somewhat similar fashion as small amplitude vibrational collective modes can model random phase approximations (RPA) correlations.

In order to implement this type of excitation on top of Hartree-Fock rotational solutions we have solved the generalized Routhian problem with two constraints ( $\Omega$  and  $\omega$ , both  $\neq 0$ ). Indeed we have made calculations for fixed values of  $I$  upon varying  $J$ . Clearly the single constraint corresponds to the yrast state (up to the quantization of  $J$  of course). It is therefore no surprise to find it at the minimum of a somewhat parabolic energy curve  $\mathcal{E}_I(J)$  as demonstrated in Fig. 2. Note in passing that the general pattern of such energy curves  $\mathcal{E}_I(J)$  for various values of  $I$  is consistent with a quadratic dependence of the total energy in  $I$  and  $J$  as assumed in Ref. [1] and calculated in the simple model case of Ref. [12].

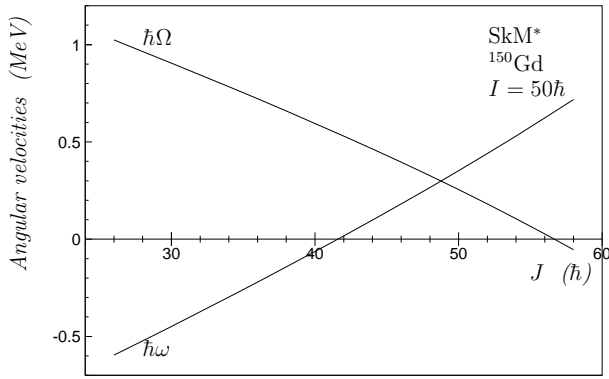


FIG. 3. Variations of the two relevant angular velocities  $\Omega$  and  $\omega$  as functions of the Kelvin circulation  $J$  for  $I = 50\hbar$  solutions.

Now to make excursions, for a given value of  $I$ , out of the yrast solution, one has to perform a two Lagrange multipliers search, whose result is exemplified in Fig. 3 for the  $I = 50\hbar$  case. It is rather significant that to get  $J$  values smaller than the yrast value, one should add a counterrotating intrinsic vortical mode. This can be explained by using the quadratic approximation for the total energy. One finds [12]

$$I = C\Omega + B\omega \quad (5a)$$

$$J = B\Omega + A\omega \quad (5b)$$

where  $A$ ,  $B$ , and  $C$  are (positive) inertia parameters defined in Ref. [12]. From the above, one finds trivially

$$J = \frac{BI}{C} + \omega \frac{AC - B^2}{C}, \quad (6)$$

where the coefficient of  $\omega$  is found to be positive for well-deformed nuclei as easily seen from the semiclassical estimates of Refs. [1,12]. Therefore one can check that starting from  $\Omega > 0$  for the yrast state, in order to decrease  $J$ , one diminishes (from its vanishing value) the  $\omega$  velocity while increasing  $\Omega$  (keeping  $I$  constant) so that one gets  $\omega\Omega < 0$ . Conversely the same reasoning yields  $\omega\Omega > 0$  when increasing  $J$  away from its yrast value.

Pairing correlations act against the global rotations (see, e.g., Ref. [27]). In our collective model account of such correlations, this corresponds to a product of angular velocities such that  $\omega\Omega < 0$ . Consequently starting from a noncorrelated (Hartree-Fock) solution, the inclusion of pairing correlations will tend to decrease the Kelvin circulation value (see Fig. 3). In Fig. 4, the dynamical moments of inertia  $\mathcal{J}^{(2)}$  are plotted for three values of  $I$  around  $50\hbar$  as functions of  $J$ . One sees that pairing correlations will indeed increase  $\mathcal{J}^{(2)}$  from its Hartree-Fock value as actually found in the Hartree-Fock-Bogoliubov calculations of Ref. [26]. In this paper, the authors have shown upon using simple yet realistic pairing matrix elements that the correlations raise the moment of inertia to the vicinity of the experimental value (typically  $\mathcal{J}^{(2)} \sim 90\hbar^2 \text{ MeV}^{-1}$ ). It is striking that when constraining the intrinsic vortical mode to obtain this value for  $\mathcal{J}^{(2)}$ , one gets a Kelvin circulation of  $\sim 25\hbar$  which fulfills precisely the  $I/J$  ratio condition of being close to 2. The latter provides a  $4\hbar$  period for the oscillating behavior of the  $\gamma$  transition energies in the  $^{150}\text{Gd}$  region.

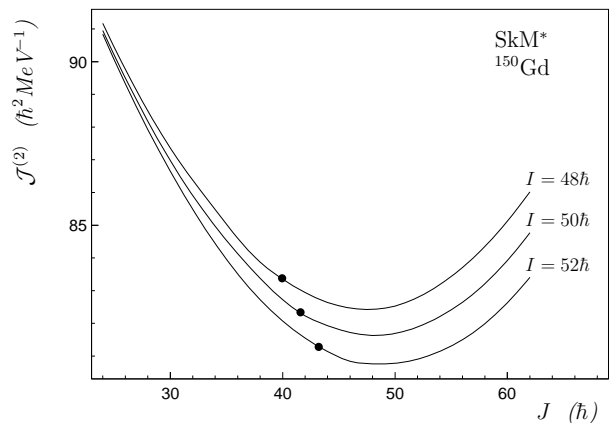


FIG. 4. Dynamical moment of inertia  $\mathcal{J}^{(2)}$  as a function of the Kelvin circulation  $J$  for angular momentum values around  $I = 50\hbar$ . Dots correspond to yrast solutions.

To confirm the above conclusion, it would be, of course, very interesting to perform variational generalized Routhian calculations within the Hartree-Fock-Bogoliubov approximation. We are currently working on it. Nevertheless, it seems to us very likely that our indirect estimate already leads to the conclusion that pairing correlations play the major role in fine-tuning the staggering period to what is experimentally observed.

## ACKNOWLEDGMENTS

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